Modeling of unsteady natural convection and thermal radiation within a saturated porous enclosure. Application for porous heat exchangers

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Abstract

We deal with a numerical study of unsteady natural convection and thermal radiation in a porous bed of large spherical particles with high emissivity confined between two-vertical hot plates and saturated by a homogeneous and isotropic fluid phase.
We aim to investigate the effects of radiative properties on fluid flow and heat transfer behavior inside the porous material. The numerical results show that the volumetric flow rate and the convective heat flux exchanged at the channel's exit are found to be increased when the particle emissivity (ε) and/or the absorption coefficient (κ) increase or when the scattering coefficient (σ_s) and/or the single scattering albedo (ω) decrease. Furthermore, the amount of heat (Q_c) transferred to fluid and the energetic efficiency E_c is found to be increased due to a raise in ε values.

Keywords: natural convection, thermal radiation, particle emissivity, absorption coefficient, scattering coefficient, single scattering albedo, porous heat exchanger.

NOMENCLATURE

A	Aspect ratio of the channel ($=H/D$)
$Bi_{i,o}$	Modified inlet (respectively, outlet) Biot Numbers ($=h_{i,o}D/\lambda$)
N	Planck number ($=\lambda\beta\Delta T/4n^2\sigma T^4$)
R	Temperature ratio ($=T_\infty/T_h$)
Ra	Modified Rayleigh number ($=kg\beta_f D\Delta T/\alpha\nu_f$)
Greek letters	
α	Thermal diffusivity, [$=\lambda/(\rho c_p)_f$] $m^2.s^{-1}$
γ	Volumetric specific heat ratio [$=(\rho c_p)_{eff}/(\rho c_p)_f$]
δ	Average porosity

I. INTRODUCTION

Transport phenomena by natural convection coupled with thermal radiation in porous media have been motivated by various applications such as thermal insulation technology, material processing, packed bed heat exchangers, to name just few applications.

Four distinct approaches are often used to estimate the equivalent radiative properties of a porous material. The first approach called the independent scattering theory is based on the knowledge of the radiative properties of each individual particles (Bohren and Huffman 1983). The second approach called the theory of multiple scattering (Tsang et al. 2000) is based on the resolution of the equation governing the propagation of electromagnetic fields, also named diffusion equation. The third approach is the inverse method of parameter identification (Baillis and Sacadura, 2000). The fourth approach is the statistical method of Monte Carlo (Tancrez and Taine 2004).

In this paper, we are interested in opaque and homogenous medium with particles of size greater than the radiation wavelength for which the multiple scattering approach is used to predict the equivalent radiative properties. The effects of equivalent radiative properties on fluid flow and heat transfer behavior inside the porous material are analysed and discussed.

II. MATHEMATICAL FORMULATION

We consider a vertical channel filled with a fluid-saturated porous medium formed by highly emissive large spherical particles, and subjected to a uniform wall hot temperature. The porous medium, at local thermal equilibrium assumption, is considered as a homogeneous, isotropic, and participating medium that can emit, absorb, and scatter isotropically radiative energy. The bounding walls of the channel, with constant emissivity ξ and reflectivity ρ, are assumed to be gray-diffuse surfaces. The Darcy flow model is assumed to be valid. The fluid is Newtonian and assumed to be a Boussinesqian one.

II. 1. Governing equations

The nondimensionalization of the governing equations is carried out on the basis of appropriate scales (Slimi et al. 2004). Hence, the dimensionless governing equations in cartesian coordinates are written as follows:

Mass conservation equation

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_z}{\partial z}=0 \qquad (1)$$

Momentum conservation equation

$$v_x=-\frac{\partial P}{\partial x}, \quad v_z=-\frac{\partial P}{\partial z}+Ra\ T \qquad (2)$$

Energy conservation equation

$$\gamma\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}-v_x\frac{\partial T}{\partial x}-v_z\frac{\partial T}{\partial z}-\frac{\tau_D^2(1\text{-}\omega)}{N}\left[\left[R+(1-R)T\right]^4-\frac{1}{4}\int_{\Omega=4\pi}I(x,z,\Omega)d\Omega\right] \qquad (3)$$

Radiative transfer equation

$$\frac{1}{\tau_D}\frac{\partial I(s,\vec{\Omega})}{\partial s}=-I(s,\vec{\Omega})+(1\text{-}\omega)\,I_b(s)+\frac{\omega}{4\pi}\int_{\Omega'=4\pi}I(s,\vec{\Omega}')d\Omega' \qquad (4)$$

II. 2. Initial and boundary conditions

Initially, we have:

$$P(x,z,0)=0\,;\ T(x,z,0)=0 \qquad (5)$$

The bounding walls (i.e., at *x=0* and *x=1*) are impermeable:

$$\left.\frac{\partial P}{\partial \mathrm{x}}\right|_{\mathrm{x}=0}=0, \quad \left.\frac{\partial P}{\partial \mathrm{x}}\right|_{\mathrm{x}=1}=0 \qquad (6)$$

At *z=0* and for lower modified Rayleigh number, the motorise pressure is assumed to be constant (Dalbert et al., 1981). At *z=A*, the jet of fluid runs into the atmosphere:

$$P(x,0,t)=0, \quad P(x,A,t)=0 \qquad (7)$$

On *x*=0 and, *x=1*, a constant hot wall temperature is imposed:

$$T(0,z,t)=1, \quad T(1,z,t)=1 \qquad (8)$$

On *z=0*, we introduce a heat transfer coefficient h_i as (Slimi et al. 2004):

$$-\frac{\partial T(x,0,t)}{\partial z}=-Bi_i T(x,0,t)+\frac{\tau_D R^4}{4N}\left\{\left[1+\left(\frac{1-R}{R}\right)T(x,0,t)\right]^4-1\right\} \qquad (9)$$

On *z=A*, a thermal boundary condition taking into consideration a reverse flow condition has been used as follows:

$$-\frac{\partial T(x,A,t)}{\partial z} = Bi_o T(x,A,t) - \frac{\tau_D R^4}{4N}\left\{\left[1+\left(\frac{1-R}{R}\right)T(x,A,t)\right]^4 - 1\right\} \quad \text{if } v_z > 0 \tag{10a}$$

$$-\frac{\partial T(x,A,t)}{\partial z} = Bi_i T(x,A,t) - \frac{\tau_D R^4}{4N}\left\{\left[1+\left(\frac{1-R}{R}\right)T(x,A,t)\right]^4 - 1\right\} \quad \text{if } v_z < 0 \tag{10b}$$

The boundary conditions for radiation intensity can be written as (Fiveland, 1988):

$$I(0,z) = \frac{\xi}{n^2\pi} + \frac{\rho}{\pi}\int_{\vec{u}.\vec{\Omega}'>0} I(0,z,\Omega')\left|\vec{u}.\vec{\Omega}'\right| d\Omega' \tag{11}$$

$$I(1,z) = \frac{\xi}{n^2\pi} + \frac{\rho}{\pi}\int_{\vec{u}.\vec{\Omega}'>0} I(1,z,\Omega')\left|\vec{u}.\vec{\Omega}'\right| d\Omega' \tag{12}$$

III. NUMERICAL METHOD

The numerical solution is based on the classical two-dimensional finite-volume method (Patankar, 1980). The Temporal derivative is discretised by a fully implicit scheme, while the convection and diffusion terms are discretised using the power-law and the central difference scheme, respectively.

The convergence criteria for the iterative procedure is as follows:

$$Max\left|\frac{\Phi_{m,n}^{i+1} - \Phi_{m,n}^{i}}{\Phi_{m,n}^{i}}\right| < 5\times 10^{-6} \tag{13}$$

where Φ stands for P and T at each point (m, n), and i is the iteration level.

The number of spatial control volumes is $M\times N=41\times 41$ and the number of control angles is $N_\theta \times N_\varphi = 6\times 8$.

The present numerical code has been validated with the most available related works. Numerical results, not shown here for the sake of brevity, show satisfactory agreement.

The effective radiative properties of the porous bed are calculated using the comprehensive approach of Singh and Kaviany (1991):

$$\kappa = \varepsilon\beta\,;\ \sigma_s = (1-\varepsilon)\beta;\ \omega = 1-\varepsilon;\ \beta = 1.5(1-\delta)Sr/d_p$$

$$S_r = 1 + 1.84(1-\delta) + 3.15(1-\delta)^2 + 7.20(1-\delta)^3 \ \text{ for } \delta > 0.3 \tag{14}$$

Where κ, β, ω, and ε are, respectively, the absorption coefficient, the extinction coefficient, the single scattering albedo, and the particle emissivity.

IV. RESULTS AND DISCUSSION

For numerical simulations, we set $A=1$, $\gamma=0.4$, $Ra=10^2$, $N=10^{-2}$, $Bi_i=10^2$ and $Bi_o=10^3$.

The volumetric flow rate q_v and the convective heat flux Q exchanged at the channel's exit. q_v and Q are written as (Slimi et al., 2004):

$$q_v = \int_0^1 v_z dx\,,\quad Q = \int_0^1 v_z T dx \tag{15}$$

Figure 1(a-b) shows that the increase in ε or κ values (or the decrease in σ_s or ω values) generates an increase in the volumetric flow rate q_v and the convective heat flux Q at the upper face of the enclosure.

Figure 2 provides, the amount of heat (Q_c) transferred to fluid with respect to the particle emissivity for various time values. In dimensional form, Q_c is written as follows:

$$Q_c = m_f c_P (T_o - T_i) \tag{16}$$

Where m_f is the mass of fluid contained in the channel. T_o and T_i are the average fluid temperatures at the exit, respectively at the entrance of the channel.

It is clear that the amount of heat (Q_c) transferred to surrounding fluid increases according to the emissivity of solid particles. In addition, Q_c is found to be increased as time goes on.

We have also calculated, at the steady state regime, the energetic efficiency E_c for various values of the particle emissivity ε. E_c is written as follows:

$$E_c = \frac{Q_c}{P_e \tau} \tag{17}$$

Where P_e denotes the electric power dissipated by the channel hot plates and τ is the due time for the establishment of the steady state regime.

The energetic efficiency E_c is calculated using industrial plates which integrate heating resistors with an electric power of about *3 kW*. This electric power value is chosen sufficiently high to be able to heat perfectly the medium.

Table 1 shows the tabulated values of the energetic efficiency, at the steady state regime, for various values of the particle emissivity. It is clear that E_c increases with ε. This increase is mainly due to the raise in the temperature values caused by the effect of thermal radiation.

E	*0*	*0.25*	*0.5*	*0.75*	*1*
E_c	*0.49*	*2.84*	*3.53*	*3.90*	*4.13*

Table 1. Tabulated values of the energetic efficiency, E_c for various values of the particle emissivity, ε at the steady state regime.

As a practical application, we propose the model of a heat exchanger which includes a porous bed constituted by spherical particles with high emissivity (*Figure 3*). The medium porosity should be sufficiently high to ensure an easy fluid circulation through the pores. In this application, each spherical particle constitutes a secondary heat source by emitting thermal radiation contributing to an additional heating of the volume of fluid surrounding each spherical particle.

V. CONCLUSION

A numerical study has been performed to investigate the effects of radiative properties on fluid flow and heat transfer by unsteady free convection and thermal radiation in a porous bed confined between two-vertical hot plates and saturated by a homogeneous and isotropic fluid phase. It has been shown that the volumetric flow rate and the convective heat flux exchanged at the channel's exit are found to be increased when the particle emissivity (ε) and/or the absorption coefficient (κ) increase or when the scattering coefficient (σ_s) and/or the single scattering albedo (ω) decrease. Furthermore, the amount of heat (Q_c) transferred to fluid and the energetic efficiency E_c are found to be increased due to a raise in the ε values.

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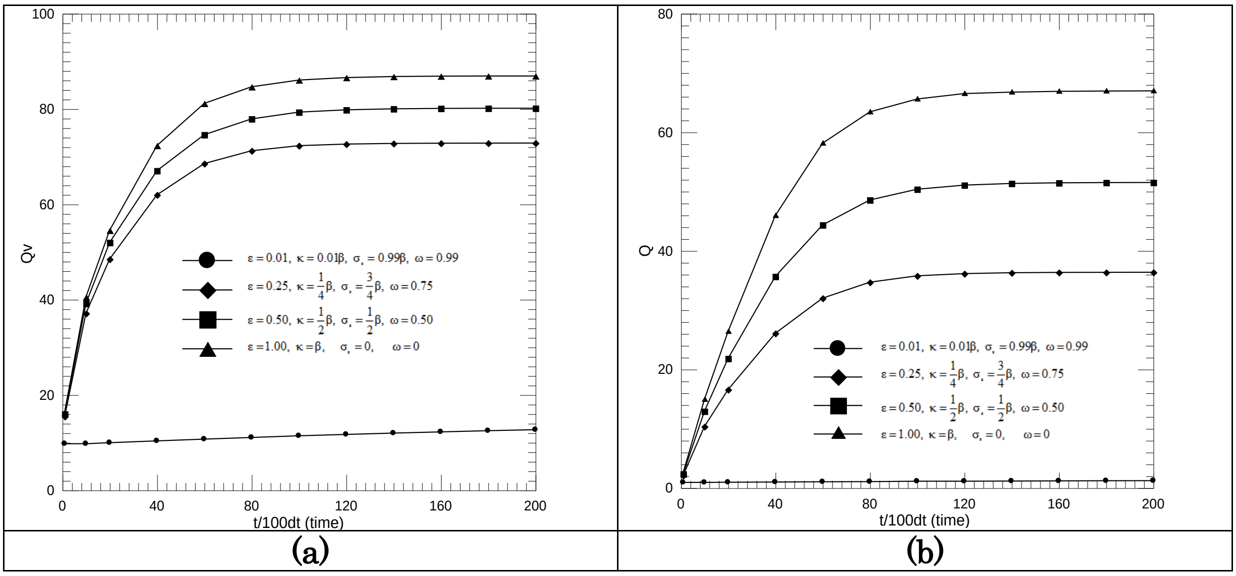


Figure 1. Time evolutions of (a) q_v and (b) Q versus ε, κ, σ_s and ω.

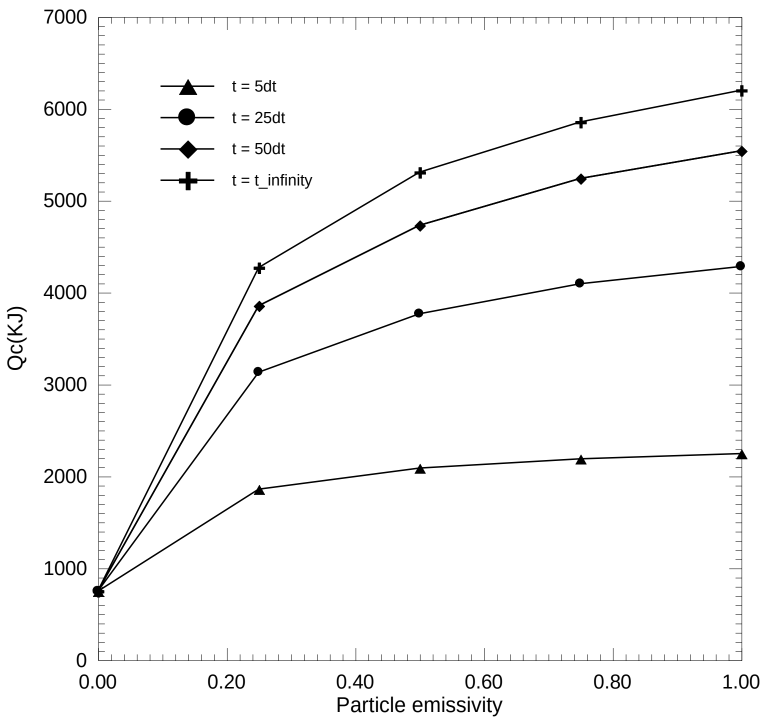
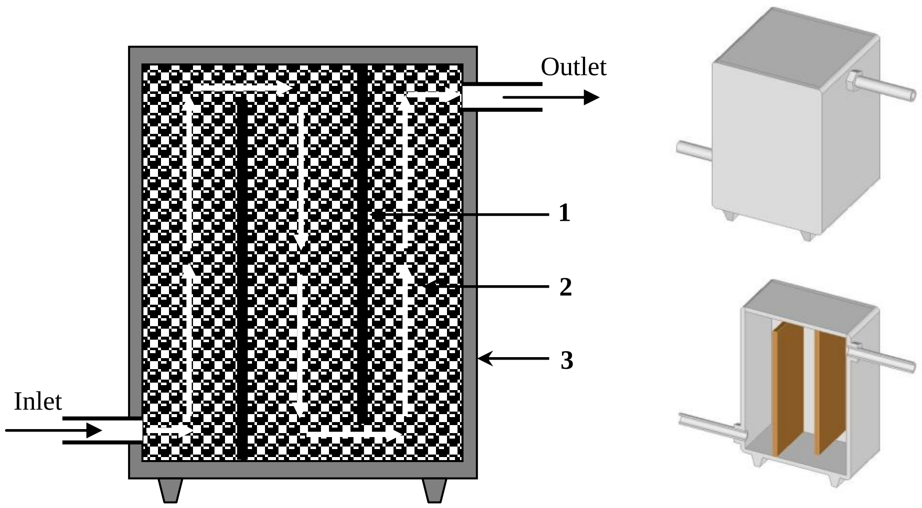


Figure 2. Amount of heat transferred to the fluid as a function of ε for various time values.



1 Hot plate **- 2** Porous bed **- 3** adiabatic cover

Figure 3. A skeleton of a heat exchanger including a porous bed formed by high emissive spherical particles