A closed loop identification software for dynamic systems: ODOE4OPE (Optimal Design Of Experiments for Online Parameter Estimation)

Jun QIAN[1,2], Pascal DUFOUR[1,3], Madiha NADRI[1]

[1]Laboratory of Process Control and Chemical Engineering (LAGEP), UMR5007, CNRS, University Claude Bernard Lyon 1
[2]Acsystème company (IT and Control engineering), Rennes, France
[1,2]Emails: jun.qian@acsysteme.com or qian@lagep.univ-lyon1.fr; dufour@lagep.univ-lyon1.fr; nadri@lagep.univ-lyon1.fr;
[3]Project leader and contact. Software website: http://odoe4ope.univ-lyon1.fr

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07/12/2012

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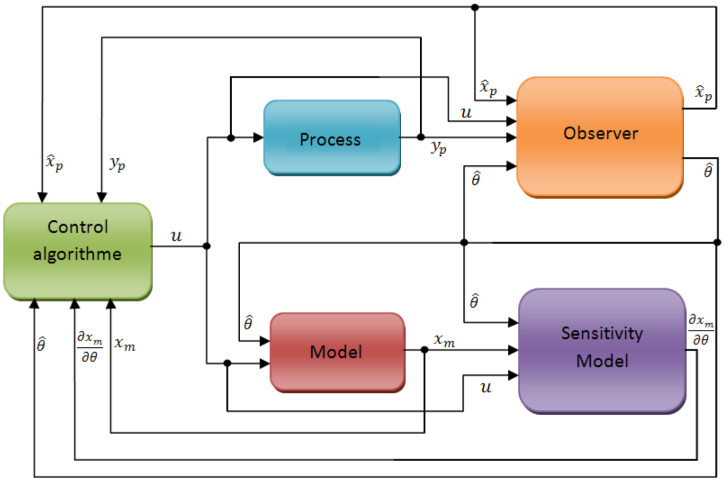
Closed loop control structure



The components

Model (linear or nonlinear)

$$(M)\left\{\begin{array}{rcl}\dot{x}(t) & = & f(x(t),\theta,u(t))\\ y(t) & = & h(x(t),\theta,u(t))\end{array}\right. \quad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^p$ is the output vector, $u \in \mathcal{U} \subset \mathbb{R}^m$ is the input vector, $\theta \in \mathbb{R}^q$ is the unknown constant parameters vector.

Observer

- system augmented with the unknown constant model parameters.
- synthesis of an observer for the system augmented: high gain observer, EKF, adaptive-gain observer, ...

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Sensitivity model

$$(M_\theta)\begin{cases} \frac{\partial}{\partial t}\tilde{x}_\theta = \frac{d}{\partial t}\left(\frac{\partial x}{\partial \theta}\right) = \frac{\partial}{\partial \theta}\left(\frac{\partial x}{\partial t}\right) = \frac{\partial f}{\partial \theta} + \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \theta} = \tilde{y}_\theta = \frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial x}\frac{\partial x}{\partial \theta} \end{cases} \quad (2)$$

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Optimal control law design

- Sensitivity matrix:

$$Z_k = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_1}\Big|_k & \frac{\partial x_1}{\partial \theta_2}\Big|_k & \cdots & \frac{\partial x_1}{\partial \theta_p}\Big|_k \\ \frac{\partial x_2}{\partial \theta_1}\Big|_k & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial x_n}{\partial \theta_1}\Big|_k & \cdots & \cdots & \frac{\partial x_n}{\partial \theta_p}\Big|_k \end{bmatrix} \quad (3)$$

- Fisher Information Matrix (FIM): $M_k = Z_k^T Z_k$
- Cost function

$$\begin{cases} J = \phi(F(x_{j|k}, u_{j|k}, \theta_{k|k})), \\ \text{with } F(x_{j|k}, u_{j|k}, \theta_{k|k}) = \frac{1}{N_p} \sum_{j=k+1}^{k+N_p} M_{j|k} \\ c_i(y, x, \theta, u) < 0 \end{cases} \quad (4)$$

- Criterion: A-optimality

$$\begin{cases} u = & arg\max_{u \in [u_{min}, u_{max}]} J_A(u) \\ \text{with} & J_A(u) = trace(F) \end{cases} \quad (5)$$

Stage of development

- History: created in 2009
- Fundamentals Aspects:
  - 2006-2010: Saida Flila's PhD thesis
  - Mars 2012: PhD thesis CIFRE (J. QIAN) between Acsystème and LAGEP (UMR5007, CNRS, UCBL1)
- User interface: under Matlab, GUI under development

The nonlinear model of Bio-reactor: $X + S \rightarrow X$

The nonlinear dynamical model of the process is:

$$(\Sigma)\begin{cases} \dot{X}(t) = \dfrac{\mu_{max} S(t)}{S(t)+K} X(t) - D(t)X(t) \\ \dot{S}(t) = -\alpha \dfrac{\mu_{max} S(t)}{S(t)+K} X(t) - D(t)\left(S(t) - S_{in}\right) \\ y(t) = X(t). \end{cases} \quad (6)$$

where:

- Inputs: a scalar controllable dilution rate $D(t)$ (h^{-1}) and an substrate concentration S_{in} (g/L).
- Output: a biomass $X(t)$ (g/L)
- Unknowns constants parameters: μ_{max} and α.
- input constraints: $0\,h^{-1} \leqslant D(j) \leqslant 0.2\,h^{-1}$
- output constraint: $X(j) \leqslant 1.95\,g/L$

Objective: based on (Σ) online identify the unknowns parameters.

Simulation results

- Input applied: $D(t)$

Simulation results

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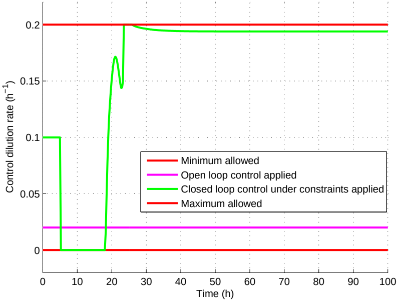
- Process output: Biomass $X(t)$



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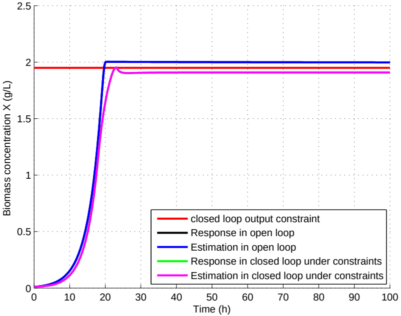

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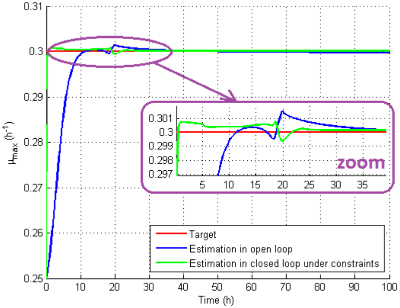

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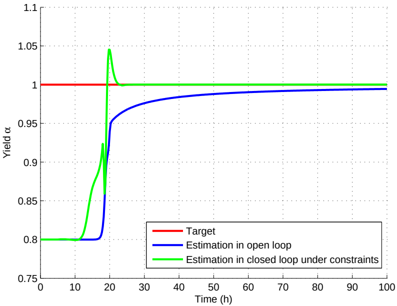

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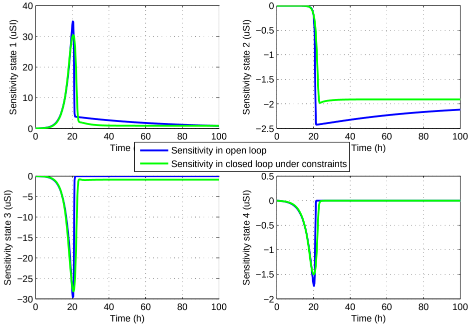

Conclusion

- ODOE4OPE is able to design online the optimal experiment under constraints.
- ODOE4OPE is able to identify online model parameters.
- The combination of an observer and a predictive control in closed loop improve the speed of the parameter estimation.
- The sensitivity criteria improve the accuracy of parameter estimation and leads to an optimal control at the same time.
- The input and output constraints specify the physical limitations imposed by the system and ensure the efficiency of the DOE.
- The software may be adapted and tuned for any user defined dynamic model.

A Model Predictive Control software: MPC@CB

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Main features of the MPC@CB software

- A control software for dynamic systems based on any kind of model: SISO or MIMO (S=single, M=multiple), linear or nonlinear, time variant or time invariant, with ordinary differential equations (ODE) and/or partial differential equations (PDE).
- A model predictive control (MPC) strategy for solving an optimal control problem (trajectory tracking, processing time minimization, any user defined criteria ...) with input constraints and with (or without) output constraints.

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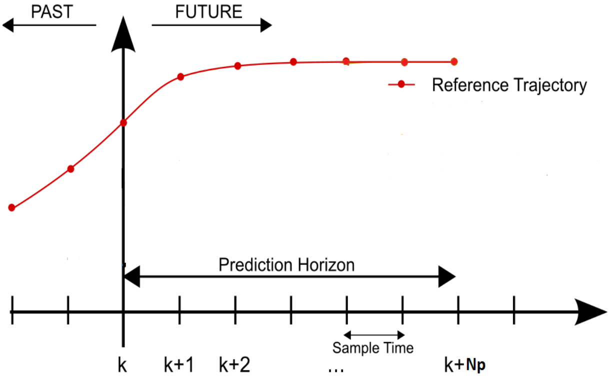
MPC: general framework

- MPC scheme



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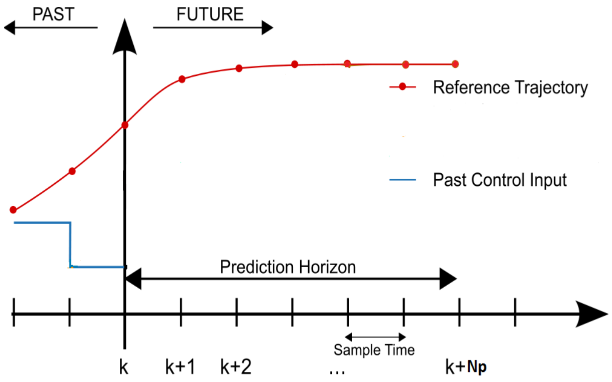

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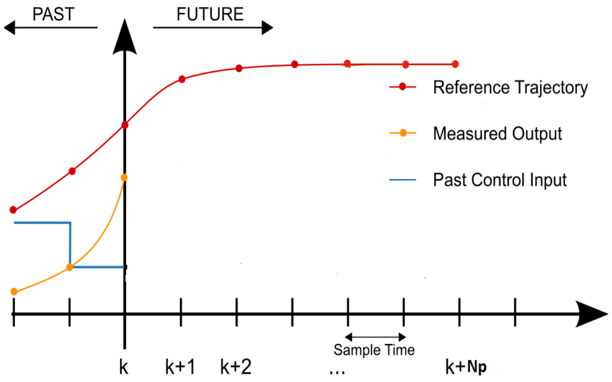

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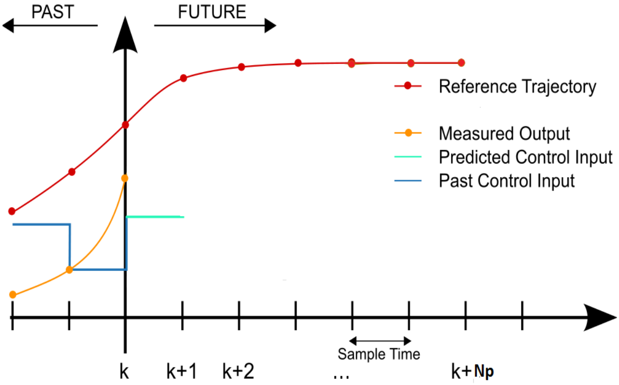

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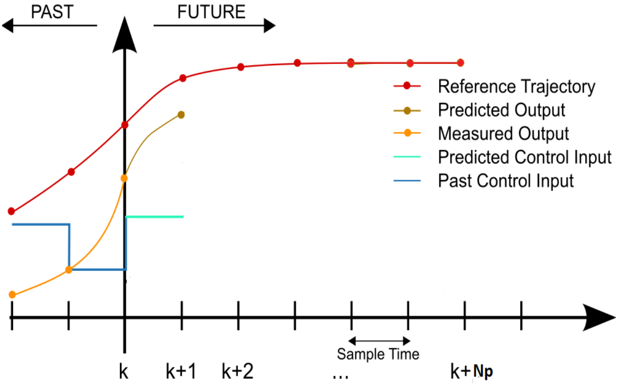

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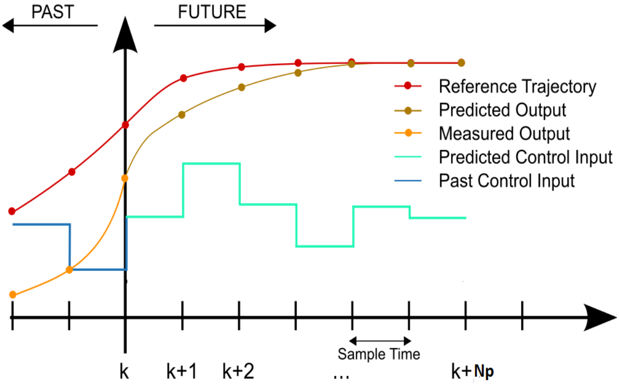
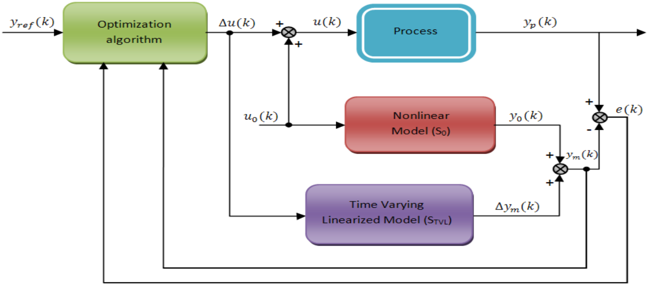


Linearized IMC-MPC structure



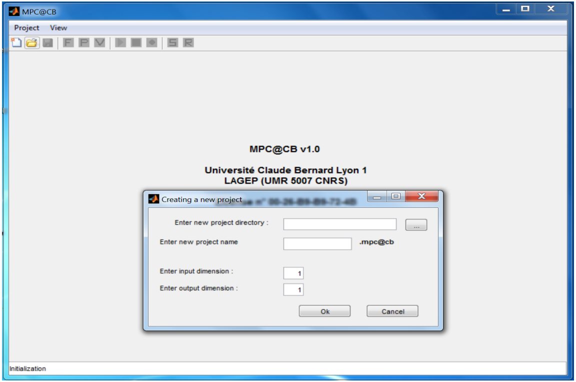
MPC@CB is based on an internal model control (IMC) structure where:

- Nonlinear model S_0 is solved off-line.
- Time-varying linearized model S_{TVL} (obtained from S_0) is solved on-line.
- Off-line open loop results are used on-line for the correct closed loop optimal constrained tuning of the control action.

Formulation of the optimization problem solved in a MPC approach

$$
\left\{
\begin{array}{l}
\min_p J_{tot} = J(p) + J_{ext}(p) \\
J(p) = \sum_{j=k+1}^{k+N_p} g(y_{ref}(j), \Delta y_m(j), \Delta u(p(j)), e(k)) \\
J_{ext}(p) = \sum_{j=k+1}^{k+N_p} (\sum_{i=1}^{N_c} w_i max^2(0, c_i(y_{ref}(j), \Delta y_m(j), \Delta u(p(j)), e(k)))) \\
p\text{: unconstrained input parameter} \\
c_i\text{: output constraints for the controlled variables} \\
\text{Input constraints handling: hyperbolic transformation} \\
\text{Ouput constraints handling: exterior penalty method} \\
\text{Control algorithm: Levenberg-Maquardt's algorithm}
\end{array}
\right.
\tag{1}
$$

Stage of development for MPC@CB



- History: created in 2007, under Matlab, with GUI
- Today: a standalone application without Matlab is available

The nonlinear model of CSTR

A continuous stirred tank reactor (CSTR): $A \rightarrow B$ is described as follows:

$$(\Sigma)\begin{cases} \dot{c}_A(t) = \frac{q}{V}(c_A^f - c_A(t)) - k_0 exp\left(-\left(\frac{E}{R}\right)/T(t)\right) c_A(t) \\ \dot{T}(t) = \frac{q}{V}(T_f - T(t)) + \frac{\Delta H}{\rho C_p} k_0 exp\left(-\left(\frac{E}{R}\right)/T(t)\right) c_A + \frac{UA}{\rho V C_p}(T_c - T(t)) \\ y(t) = c_A(t) \end{cases} \tag{2}$$

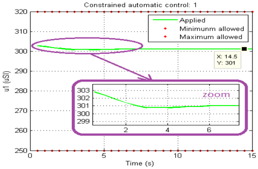
Where:

- Input: the controllable temperature of cooling jacket $T_c(t)$ (K).
- Output: the concentration of A $c_A(t)$ (mol/m^3)
- Input constraints: $250K < T_c < 320K$.
- Output constraint $y > y_{min} = 0.87$.

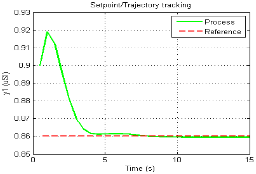
Objective: use MPC@CB (with or without the output constraint) for the set-point tracking of a reference value 0.86.

Simulation results

- without the output constraint
  - optimal input applied

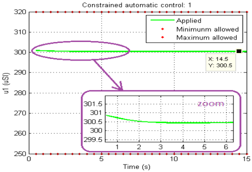


  - setpoint trajectory tracking

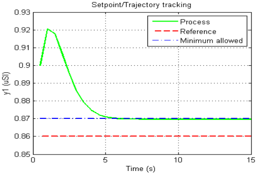


Conclusion: setpoint regulation, OK!

- with the output constraint: $y > y_{min} = 0.87$
  - optimal input applied



  - trajectory tracking



Conclusion: constrained setpoint regulation, OK!

Conclusion

- MPC@CB is easily tunable for any new dynamic process.
- The specified user defined constrained control objectives are well achieved by the online closed loop control with MPC@CB.
- With the off-line and on-line IMC-MPC structure, the on-line computational time of optimization is decreased by MPC@CB.
- More case studies are discussed on the website.
- MPC@CB is available: short time evaluation, commercial licence or embedded in a complete turnkey solution for the customer.

Contact for the softwares

Pascal DUFOUR

Associate professor
University Claude Bernard Lyon 1, Campus de la Doua
Ecole CPE, bât 308G, étage 3
LAGEP UMR 5007, Bureau G322
3 rue Victor Grignard
69100 Villeurbanne, France
Tel: +33 4 72 43 18 78
dufour@lagep.univ-lyon1.fr
http://www.tinyurl.com/dufourpascal

ODOE4OPE

Email: odoe4ope@univ-lyon1.fr
Website: http://odoe4ope.univ-lyon1.fr

MPC@CB

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References

ODOE4OPE

[1] S. Flila, P. Dufour and H. Hammouri, *Identification optimale en boucle fermée pour les systèmes non linéaires*, 6th IEEE Conference International Francophone Automatic (CIFA), Nancy, France, 2012.

[2] S. Flila, P. Dufour, H. Hammouri and Nadri M., *A combined closed loop optimal design of experiments and online identification control approach*,in 29th IEEE C 2010, Beijing, China.

MPCatCB

[1] N. Daraoui, P. Dufour, H. Hammouri, A. Hottot, *Model predictive control during the primary drying stage of lyophilisation*, Control Engineering Practice, 2010, 18(5), pp. 483-494.

[2] I. Bombard, B. Da Silva, P. Dufour and P. Laurent, *Experimental predictive control of the infrared cure of a powder coating: a non-linear distributed parameter model based approach*, Chemical Engineering Science Journal, 2010, 65(2), pp. 962-975.

[3] P. Dufour, Y. Touré, D. Blanc, P. Laurent,*On nonlinear distributed parameter model predictive control strategy: On-line calculation time reduction and application to an experimental drying process*, Computers and Chemical Engineering, 2003, 27(11), pp.1533-1542.

Annex A: ODOE4OPE

- Simulation condition
  - Parameters in the model of the bio-reactor

Parameter	Symbol	Value
The maximal specific rate of the biomass (h^{-1})	μ_{max}	0.3
The yield (-)	α	1
The constant of the saturation (g/L)	K	0.05
The substrate concentration in the feed (g/L)	S_{in}	2

  - Initial conditions and parameters value for the simulation

Initial conditions and Parameters	Symbol	Value (Unit)
Target values of parameters	$[\theta_1\ \theta_2]_p$	[0.3 1]
Initial estimates of parameters	$[\hat{\theta}_1(0)\ \hat{\theta}_2(0)]$	[0.25 0.8]
Initial values of model states	$[x_{m1}(0)\ x_{m2}(0)]$	[0.01 2]
Initial estimates of states	$[\hat{x}_1(0)\ \hat{x}_2(0)]$	[0.01 1.5]
Initial estimate of covariance	$P(0)$	$50 \times I$
Time of the simulation	T_{fin}	100 h
Sampling period	T_s	0.25 h
Prediction horizon	N_p	8

Annex A: ODOE4OPE

- Observer for bio-reactor
  - System augmented:

$$(M)\begin{cases} \dot{x}_1(t) = \dfrac{\theta_1 x_2(t)}{x_2(t)+a_1} x_1(t) - u(t)x_1(t) \\ \dot{x}_2(t) = -\theta_2 \dfrac{\theta_1 x_2(t)}{x_2(t)+a_1} x_1(t) - u(t)\left(x_2(t) - a_2\right) \\ \dot{\theta}_1 = 0 \\ \dot{\theta}_2 = 0 \\ y(t_k) = x_1(t_k), \end{cases} \tag{3}$$

where $t_k - t_{k-1}$ is the sampling time measurements.

Annex A: ODOE4OPE

- Extended Kalman Filter (EKF)

Model	$\dot{x}(t) = f(x(t), u(t)) + w(t), w(t) \in N(0, Q(t))$
	$y_k = h(x_k) + v_k, v_k \in N(0, R_k)$ where $x_k = x(t_k)$
Initialize	$\hat{x}_{0\|0} = E[x(t_0)],\ P_{0\|0} = Var[x(t_0)]$
Predict	$\begin{cases} \dot{\hat{x}}(t) = f(\hat{x}(t), u(t)) \\ \dot{P}(t) = F(t)P(t) + P(t)F(t)^T + Q(t) \end{cases}$
	with $\begin{cases} \hat{x}(t_{k-1}) = \hat{x}_{k-1\|k-1} \\ P(t_{k-1}) = P_{k-1\|k-1} \end{cases}$
	$\Rightarrow \begin{cases} \hat{x}_{k\|k-1} = \hat{x}(t_k) \\ P_{k\|k-1} = P(t_k) \end{cases}$
Update	$K_k = P_{k\|k-1}H_k^T(H_kP_{k\|k-1}H_k^T + R_k)^{-1}$
	$\hat{x}_{k\|k} = \hat{x}_{k\|k-1} + K_k(y_k - h(\hat{x}_{k\|k-1}))$
	$P_{k\|k} = (I - K_kH_k)P_{k\|k-1}$
	where $F(t) = \frac{\partial f}{\partial x}\Big\|_{\hat{x}(t),u(t)},\ H(t) = \frac{\partial h}{\partial x}\Big\|_{\hat{x}_{k\|k-1}}$

Annex A: ODOE4OPE

- Sensitivity model of the bio-reactor

$$\begin{cases} \dot{x}_{1\theta 1}(t) = \dfrac{x_2(t)x_1(t)}{x_2(t)+a_1} - u(t)x_{1\theta 1}(t) + \hat{\theta}_1(t)\dfrac{x_2(t)x_{1\theta 1}(t)(x_2(t)+a_1)+a_1x_1(t)x_{2\theta 1}(t)}{(x_2(t)+a_1)^2} \\ \dot{x}_{1\theta 2}(t) = -u(t)x_{1\theta 2} + \hat{\theta}_1(t)\dfrac{x_2(t)x_{1\theta 2}(t)(x_2(t)+a_1)+a_1x_1(t)x_{2\theta 2}(t)}{(x_2(t)+a_1)^2} \\ \dot{x}_{2\theta 1}(t) = -\dfrac{\hat{\theta}_2(t)x_2(t)x_1(t)}{x_2(t)+a_1} - u(t)x_{2\theta 1}(t) - \hat{\theta}_1(t)\hat{\theta}_2(t)\dfrac{x_2(t)x_{1\theta 1}(t)(x_2(t)+a_1)+a_1x_1(t)x_{2\theta 1}(t)}{(x_2(t)+a_1)^2} \\ \dot{x}_{2\theta 2}(t) = -\dfrac{\hat{\theta}_1(t)x_2(t)x_1(t)}{x_2(t)+a_1} - u(t)x_{2\theta 2}(t) - \hat{\theta}_1(t)\hat{\theta}_2(t)\dfrac{x_2(t)x_{1\theta 2}(t)(x_2(t)+a_1)+a_1x_1(t)x_{2\theta 2}(t)}{(x_2(t)+a_1)^2} \\ x_{1\theta 1}(0) = x_{1\theta 2}(0) = x_{2\theta 1}(0) = x_{2\theta 2}(0) = 0 \end{cases} \tag{4}$$

where $x_{i\theta_j} = \frac{\partial x_i}{\partial \theta_j}$.

Annex B: MPC@CB

- The constrained optimization problem based on IMC-MPC structure is described as followed:

$$\left\{\begin{array}{l} \min_{\tilde{u}} J(\tilde{u}) = \sum_{j \in \mathcal{J}_1^{N_p}} g(y_{ref}(j), \Delta y_m(j), \Delta u(j-1), e(k)) \\ \Delta \tilde{u} = [\cdots \Delta u(j) \cdots]^T \quad \forall j \in \mathcal{J}_0^{N_c-1} \\ \Delta u(j) = \Delta u(k+N_c-1) \quad \forall j \in \mathcal{J}_{N_c}^{N_p-1} \\ u_{min} - u_0(j) \leq \Delta u(j) \leq u_{min} - u_0(j) \quad \forall j \in \mathcal{J}_0^{N_p-1} \\ \Delta u'_{min} \leq \Delta u(j) - \Delta u(j-1) \leq \Delta u'_{max} \quad \forall j \in \mathcal{J}_0^{N_p-1} \\ \Delta u'_{min} = \Delta u_{min} - (u_0(j) - u_0(j-1)) \quad \forall j \in \mathcal{J}_0^{N_p-1} \\ \Delta u'_{max} = \Delta u_{max} - (u_0(j) - u_0(j-1)) \quad \forall j \in \mathcal{J}_0^{N_p-1} \\ c_i(y_{ref}(j), \Delta y_m(j), \Delta u(j-1), e(k)) \leq 0 \; \forall j \in \mathcal{J}_0^{N_p}, \; \forall i \in \mathcal{I}_1^n \\ \text{and subjet to the resolution of the model } (S_{TVL}). \end{array}\right. \tag{5}$$

Annex B: MPC@CB

- Input constraints handling: hyperbolic transformation:

$$\begin{cases} u(j) = f(p(j)) = f_{moy} + f_{amp} tanh(\frac{p(j) - f_{moy}}{f - amp}) \quad \forall j \in \mathcal{J}_0^{N_c - 1} \\ p(j) \in \mathbb{R} \; \forall j \in \mathcal{J}_0^{N_c - 1} \text{ (unconstrained input parameter)} \\ f_{moy} = \frac{f_{max} + f_{min}}{2} \\ f_{amp} = \frac{f_{max} - f_{min}}{2} \\ f_{min} = \max(u_{min}, u(j-1) + \Delta u_{min}) \quad \forall j \in \mathcal{J}_0^{N_c - 1} \\ f_{max} = \max(u_{max}, u(j-1) + \Delta u_{max}) \quad \forall j \in \mathcal{J}_0^{N_c - 1} \end{cases} \tag{6}$$

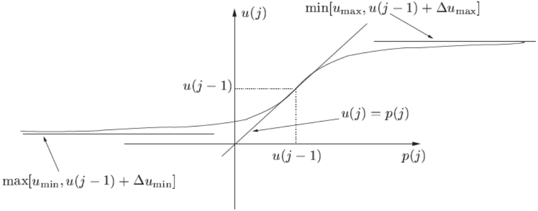

Fig. Mapping from unconstrained variable p into constrained variable u

Annex B: MPC@CB

- Control algorithm: Levenberg-Maquardt

$$\Delta\tilde{p}^{n+1} = \Delta\tilde{p}^{n} - (\bigtriangledown^2 J_{tot}^{n} + \lambda I)^{-1} \bigtriangledown J_{tot}^{n} \tag{7}$$

where the argument $\Delta\tilde{p}$ is determined at each sample instant k by this iteration procedure, $\bigtriangledown^2 J_{tot}^{n}$ and $\bigtriangledown J_{tot}^{n}$ are the criteria gradient and criteria hessain with respect to $\Delta\tilde{p}^{n}$ at the iteration n.